3/20/18

"The question isn't 'Who is going to let me?'; It's 'Who is going to stop me?'." -Ayn Rand

HW: "Logarithmic Functions" homework section
Test 3 on Wednesday 3/28

AIM: What is a Logarithmic Function?

Warm Up: (Do it on the back of your packet)

1) If the graph of the exponential function passes through the points (0,9) and (4, 16/9), what is the equation of the exponential function?

y-int
therefore
$a = 9$

$y = ab^x$

$\frac{\frac{16}{9}}{9} = \frac{9(b)^4}{9}$

$\frac{16}{81} = b^4$

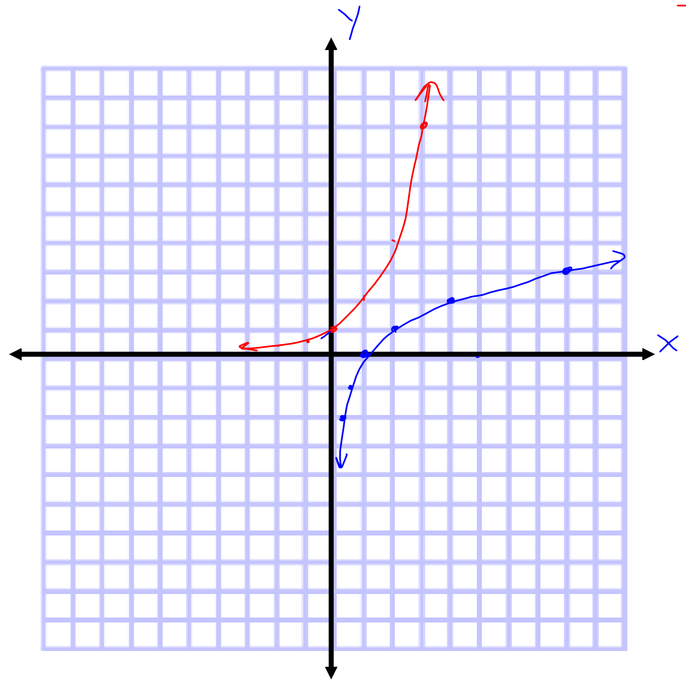
$b = \frac{2}{3}$

$\boxed{y = 9\left(\frac{2}{3}\right)^x}$

✱ The y-int of $y = ab^x$ is a.

One of the properties of the exponential function $f(x) = b^x$ is that it is a 1-1 function. Remember that this means it has an inverse function whose graph can be obtained by reflecting the graph of $y = b^x$ through the line $y = x$.

Let's graph the inverse of the function that we were looking at yesterday, $y = 2^x$.



$y = 2^x$

-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

Inverse

$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3

Write an equation for the inverse of $y = 2^x$.

Switch x and y

$\boxed{x = 2^y}$

$y = \log_b x$

The equation $x = b^y$ tells us that y is the exponent on b that produces x. In situations like this the word logarithm is used in place of exponent. A **logarithm** is an exponent. We can abbreviate to:

$$y = \log_b x$$

* log is an exponent

read as "y equals log x to the base b" or "y equals log x base b."

base

answer

exponent

Exponential form: $x = b^y$

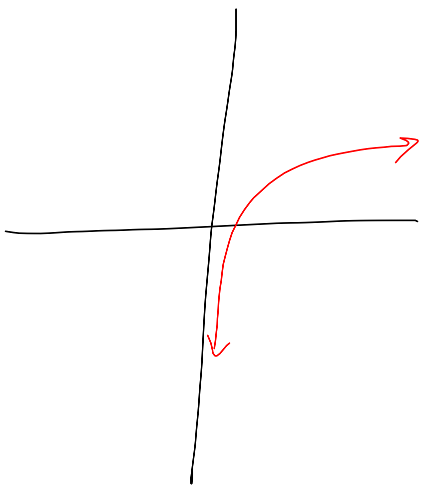
Logarithmic form: $y = \log_b x$

answer

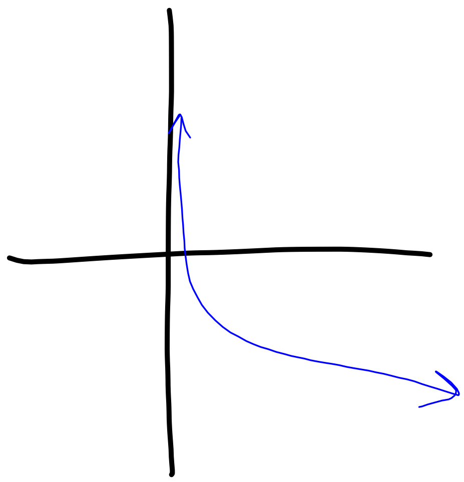
exponent

base

General Graphs of Logarithmic Functions



$y = \log_b x, b > 1$



$y = \log_b x, 0 < b < 1$

Properties of Logarithmic Functions

1. The domain consists of all positive numbers.
2. The range consists of all real numbers.
3. The function is increasing (the curve is rising) when $b > 1$, and its is decreasing (the curve is falling) when $0 < b < 1$.
4. It is a one-to-one function.
5. The point (1,0) is on the curve.
6. There is no y-intercept.
7. The y-axis is a vertictal asymptote to the curve.

Practice

1. Evaluate the following logarithms. If needed, write an equivalent exponential equation. (Without the use of your calculator.)

(a) $\log_2 8 = x$

"exponent of 2 that gets us 8."

$2^x = 8$

$2^x = 2^3$

$x = 3$

(b) $\log_4 16 = x$

$4^x = 16$

$x = 2$

(c) $\log_5 625 = x$

$5^x = 625$

$x = 4$

(d) $\log_{10} 100,000 = x$

$10^x = 100000$

$x = 5$

(e) $\log_6 \left(\frac{1}{36}\right) = x$

$6^x = \frac{1}{36}$

$x = -2$

(f) $\log_2 \left(\frac{1}{16}\right) = x$

$2^x = \frac{1}{16}$

$x = -4$

(g) $\log_5 \sqrt{5} = x$

$5^x = \sqrt{5}$

$5^x = 5^{\frac{1}{2}}$

$x = \frac{1}{2}$

(h) $\log_3 \sqrt[5]{9} = x$

$3^x = \sqrt[5]{9}$

$3^x = 9^{\frac{1}{5}}$

$3^x = (3^2)^{\frac{1}{5}}$

$3^x = 3^{\frac{2}{5}}$

$x = \frac{2}{5}$

It is critically important to understand that logarithms **give exponents as their outputs**. We will be working for multiple lessons on logarithms and a basic understanding of their inputs and outputs is critical.

2. If the function $y = \log_2(x+8)+9$ was graphed in the coordinate plane, which of the following would represent its y-intercept?

$\rightarrow x=0$

(1) 12 (3) 8

(2) 13 (4) 9

$y = \log_2(0+8)+9$

$y = \log_2(8)+9$

$y = 3+9$

$y = 12$

3. Between which two consecutive integers must $\log_3 40$ lie?

~~(1)~~ 1 and 2 (3 9)

~~(2)~~ 2 and 3 (9 27)

(3) 3 and 4 (27 81)

(4) 4 and 5

Exponent of 3 that gets me 40.

Calculator Use and Logarithms – Most calculators only have two logarithms that they can evaluate directly. One of them, $\log_{10} x$, is so common that it is actually called the **common log** and typically is written without the base 10.

$\log x = \log_{10} x$ (The Common Log)

4. Evaluate each of the following using your calculator.

(a) $\log 100 = 2$

(b) $\log\left(\frac{1}{1000}\right) = -3$

(c) $\log\sqrt{10} = .5$

5. Can the value of $\log_2(-4)$ be found? What about the value of $\log_2 0$? Why or why not? What does this tell you about the domain of $\log_b x$?

There is no exponent that we can raise 2 to in order to get -4.

There is no power we can raise 2 to in order to get 0.

Domain: $x > 0$